Section 5-4, Mathematics 104

Factoring Polynomials

We started learning to factor trinomials, polynomials of the form

$ax^2 + bx + c$

One method was to identify one of the important patterns:

$$A^2 + 2AB + B^2 = (A+B)^2$$
$$A^2 - 2AB + B^2 = (A-B)^2$$
$$A^2 - B^2 = (A+B)(A-B)$$

Example:

$4x^2 - 9 = (2x)^2 - (3)^2 = (2x+3)(2x-3)$

Another method was reverse FOIL

Example:

$2x^2 - x - 3$

$(2x\,?1)(x\,?3)\ or\ (2x\,?3)(x\,?1)$

Since we have -3 we will have one + and one -.

The first can be either 6 or -6. The second can be 1 or -1, but of course -1 is correct so

$2x^2 - x - 3 = (x+1)(2x-3)$

Student Example:

Let's try one: $3x^2 + 3x - 6$

Dividing Polynomials

There are a number of circumstances in which we will want to divide polynomials.

For example, we might have a polynomial:

$$x^3 + 6x^2 + 11x + 6$$

which we know has a factor $x + 1$

So we want to know what

$$\frac{x^3 + 6x^2 + 11x + 6}{x + 1} \text{ is.}$$

We have two ways of dividing polynomials

Long Division

and

Synthetic Division

Long Division always works.

The first is very similar to long division of numbers.

Here's how it works.

$$\begin{array}{r l} & x^2 \\ x+1 \big) & \overline{x^3 + 6x^2 + 11x + 6} \end{array}$$

We see here that the x in x+1 can be divided into x^3, x^2 times.
As with long division of numbers we multiply x^2 by x+1 and subtract

$$\begin{array}{r l} & x^2 \\ x+1 \big) & \overline{x^3 + 6x^2 + 11x + 6} \\ & \underline{x^3 + x^2} \\ & \phantom{x^3 + {}} 5x^2 + 11x + 6 \end{array}$$

We repeat the process

$$
\begin{array}{r}
x^2 + 5x + 1 \phantom{{}+6} \\
x+1 \overline{\big)\, x^3 + 6x^2 + 11x + 6} \\
\underline{x^3 + x^2} \phantom{{}+11x+6} \\
5x^2 + 11x + 6 \\
\underline{5x^2 + 5x} \phantom{{}+6} \\
6x + 6 \\
\underline{6x + 6} \\
0
\end{array}
$$

So we have $\dfrac{x^3 + 6x^2 + 11x + 6}{x+1} = x^2 + 5x + 1$

This version of division will always work.

There could be a remainder:

Example:

$$\frac{2x^2+3x+4}{x+2}$$

$\begin{array}{r} 2x \\ x+2 \overline{)2x^2+3x+4} \end{array}$	$\begin{array}{r} 2x \\ x+2 \overline{)2x^2+3x+4} \\ \underline{2x^2+4x} \\ -x+4 \end{array}$
$\begin{array}{r} 2x \\ x+2 \overline{)2x^2+3x+4} \\ 2x^2+4x \end{array}$	$\begin{array}{r} 2x-1 \\ x+2 \overline{)2x^2+3x+4} \\ \underline{2x^2+4x} \\ -x+4 \\ \underline{-x-2} \\ 6 \end{array}$

$$\frac{2x^2+3x+4}{x+2} = 2x-1+\frac{6}{x+2}$$

or
$2x$ - 1 Remainder 6

Student Example:

$$\frac{6x^2+10x+6}{2x-2}$$

Synthetic Division

Synthetic division does not do anything better than long division.
It only works if you are dividing by x-a where a is some constant.
It is also a strange looking skill that you have to learn carefully.
So why learn synthetic division?

Because once you learn it is easy.

Here's the previous example using synthetic division

$$\frac{2x^2+3x+4}{x+2}$$

Note that: x+2 = x-(-2) so the divisor number is -2

First bring down the first number

$$\begin{array}{r|rrr} -2 & 2 & 3 & 4 \\ & & & \\ \hline & 2 & & \end{array}$$

Multiply -2 by 2 and add

$$\begin{array}{r|rrr} -2 & 2 & 3 & 4 \\ & & -4 & \\ \hline & 2 & -1 & \end{array}$$

Now multiply -2 by -1 and add

$$\begin{array}{r|rrr} -2 & 2 & 3 & 4 \\ & & -4 & 2 \\ \hline & 2 & -1 & 6 \end{array}$$

The first two numbers in the result are coefficients and the last number is the remainder, so we get

2x-1 R 6

Note: It can also work in a situation like this:

$$\frac{8x^2+6x-4}{2x+4}$$

First we have to remove the coefficient from *x* in the divisor, but this is easy.

$$\frac{8x^2+6x-4}{2x+4}=\frac{8x^2+6x-4}{2(x+2)}=\frac{4x^2+3x-2}{x+2}$$

Now we can proceed as before:

$$\begin{array}{r|rrr} -2 & 4 & 3 & -2 \\ & & -4 & 2 \\ \hline & 2 & -1 & 0 \end{array}$$

So $\dfrac{8x^2+6x-4}{2x+4}=2x-1$

Student Example:

Try this: $\dfrac{3x^2-5x+8}{x-1}$